

The scalar bi-spectrum during preheating in single field inflationary models

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In single field inflationary models, preheating refers to the phase that immediately follows inflation, but precedes the epoch of reheating. During this phase, the inflaton typically oscillates at the bottom of its potential and gradually transfers its energy to radiation. At the same time, the amplitude of the fields coupled to the inflaton may undergo parametric resonance and, as a consequence, explosive particle production can take place. A priori, these phenomena could lead to an amplification of the super-Hubble scale curvature perturbations which, in turn, would modify the standard inflationary predictions. However, remarkably, it has been shown that, although the Mukhanov-Sasaki variable does undergo narrow parametric instability during preheating, the amplitude of the corresponding super-Hubble curvature perturbations remain constant. Therefore, in single field models, metric preheating does not affect the power spectrum of the large scale perturbations. In this article, we investigate the corresponding effect on the scalar bi-spectrum. Using the Maldacena's formalism, we analytically show that, for modes of cosmological interest, the contributions to the scalar bi-spectrum as the curvature perturbations evolve on super-Hubble scales during preheating is completely negligible. Specifically, we illustrate that, certain terms in the third order action governing the curvature perturbations which may naively be expected to contribute significantly are exactly canceled by other contributions to the bi-spectrum. We corroborate selected analytical results by numerical investigations. We conclude with a brief discussion of the results we have obtained.

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I. INTRODUCTION

Inflation is currently considered the most attractive paradigm for explaining the extent of homogeneity of the observable universe. In addition, the paradigm also provides a well motivated causal mechanism for the generation of perturbations in the early universe (see any of the following texts [1–7] or one of the following reviews [8–20]). In the standard picture, at the end of the inflationary epoch, the inflaton, which is coupled to the other fields of the standard model, decays into relativistic particles thereby transferring its energy to radiation (see, for instance, Refs. [12, 18] and also Refs. [21–24]). These decay products are then expected to thermalize [25] in order for the radiation dominated epoch corresponding to the conventional hot big bang model to start.

In many models, inflation is terminated when the scalar field has rolled down close to a minimum of the potential. Thereafter, the scalar field usually oscillates at the bottom of the potential with an ever decreasing amplitude. There exists a period during this regime, im-

mediately after inflation but prior to the epoch of reheating, a phase that is often referred to as preheating [26–28]. During this brief phase, as in the inflationary era, the scalar field continues to remain the dominant source that drives the expansion of the universe. Though the modes of cosmological interest (corresponding to comoving wavenumbers k such that, say, $10^{-4} < k < 1 \text{ Mpc}^{-1}$) are well outside the Hubble radius during this phase, the conventional super-Hubble solutions to the curvature perturbations that are applicable during inflation do not a priori hold at this stage. In fact, careful analysis is required to evolve these modes during the phase of preheating. However, despite the subtle effects that need to be accounted for, it can be shown that, in single field inflationary models, the amplitude of the curvature perturbations and, hence, the amplitude as well as the shape of the scalar power spectrum associated with the scales of cosmological interest remain unaffected by the process of preheating (for the original effort, see Ref. [29]; for more recent discussions, see Refs. [30–32]).

Over the last decade, there has been a tremendous theoretical interest in understanding the extent of the non-Gaussianities that are generated during inflation [33–57]. Simultaneously, there has been a constant effort to arrive at increasingly tighter constraints on the dimensionless non-Gaussianity parameter f_{NL} that is often used to characterize the amplitude of the reduced scalar bi-

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spectrum (viz. a suitable ratio of the scalar bi-spectrum to the corresponding power spectrum) from the available Cosmic Microwave Background (CMB) data [58–76]. For instance, it has been theoretically established that slow roll inflation driven by the canonical scalar field typically leads to rather small values of f_{NL} (of the order of the first slow roll parameter) [33–35, 37]. In contrast, though a Gaussian primordial perturbation lies well within 2σ , the mean values of f_{NL} from the CMB observations seem to indicate a significant amount of non-Gaussianity [58–76]. Such large levels for the parameter f_{NL} can be generated when one considers non-canonical scalar fields [38–42] or when there exist deviations from slow roll inflation [44–52].

We mentioned above that, in single field models, the epoch of preheating does not affect the curvature perturbations and the scalar power spectrum generated during inflation on cosmologically relevant scales. Note that, the scalar power spectrum is essentially determined by the amplitude of the curvature perturbation. Whereas, as we shall discuss, the scalar bi-spectrum generated during inflation involves integrals over the curvature perturbations as well as the slow roll parameters [37–42, 44–52]. If indeed deviations from slow roll inflation can result in high levels of non-Gaussianity, then, naively, one may imagine that the termination of inflation and the regime of preheating—both of which involve large values for the slow roll parameters—can also lead to large non-Gaussianities. In other words, it may seem that the epoch of preheating can contribute significantly to the scalar bi-spectrum. In this work, we shall investigate the contributions to the scalar bi-spectrum during preheating in single field inflationary models. Remarkably, though the epoch of preheating actually amplifies specific contributions to the bi-spectrum, as we shall illustrate, certain cancellations arise that leave the total bi-spectrum generated during inflation virtually unaltered.

This paper is organized as follows. In the following section, we shall highlight the essential aspects of preheating in single field inflationary models. In particular, we shall discuss the behavior of the scalar field as well as the scale factor, when a canonical scalar field is oscillating at the bottom of an inflationary potential which behaves quadratically near its minimum. We shall also discuss a few important points concerning the evolution of the curvature perturbation on super-Hubble scales during preheating. In Sec. III, after defining the bi-spectrum, we shall outline the various contributions to the bi-spectrum in the Maldacena formalism. In Sec. IV, we shall evaluate the contributions to the bi-spectrum for super-Hubble modes as the scalar field oscillates in the quadratic potential, and show that the total contribution during this epoch proves to be insignificant for these modes. We shall also support certain analytical results with the corresponding numerical computations. Finally, in Sec. V, we shall conclude with a brief summary and outlook.

A couple of words on our notation are in order at this stage of our discussion. We shall work with units such

that $c = \hbar = 1$ and shall set $M_{\text{Pl}}^2 = (8\pi G)^{-1}$. An overdot and an overprime shall denote differentiation with respect to the cosmic time t and the conformal time η , respectively. Also, N shall represent the number of e-folds. Moreover, double angular brackets shall denote averaging over the oscillations during preheating.

II. BEHAVIOR OF THE BACKGROUND AND THE LARGE SCALE PERTURBATIONS DURING PREHEATING

In this section, we shall discuss the behavior of the background and the evolution of the curvature perturbation on super-Hubble scales during preheating. We shall consider a model involving the canonical scalar field and assume that the inflationary potential behaves quadratically around its minimum.

A. Background evolution about a quadratic minimum

Consider a canonical scalar field ϕ that is governed by the quadratic potential $V(\phi) = m^2 \phi^2/2$ near its minimum. It is well known that, in such cases, slow roll inflation can be realized if the field starts sufficiently far away from the minimum, with suitably small values for its velocity [1–20]. Provided the initial conditions fall in the basin of the inflationary attractor, the number of e-folds of inflation achieved largely depends only on the initial value of the field, and inflation ends as the field nears the bottom of the potential. In fact, according to the slow roll approximation, in an inflationary potential that consists of no terms other than the above-mentioned quadratic one, inflation gets terminated as the field crosses $\phi_e = \sqrt{2} M_{\text{Pl}}$. Thereafter, the scalar field oscillates about the minimum with a constantly decreasing amplitude because of the friction caused due to the expansion. These behavior are clearly evident from Fig. 1, where we have plotted the evolution of the scalar field and the first slow roll parameter ϵ_1 , arrived at numerically, both during and immediately after inflation for the quadratic potential. We should emphasize here that focusing on single field models can be considered to be essentially equivalent to assuming that the coupling of the inflaton to other fields is suitably weak during preheating. The weak coupling will allow the condensate to live sufficiently long for a few oscillations to take place about the minimum of the inflaton potential.

Recall that, the first slow roll parameter is given by

$$\epsilon_1 = -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2 M_{\text{Pl}}^2}, \quad (1)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter, with $a(t)$ being the scale factor associated with the Friedmann-Lemaître-Robertson-Walker line-element. The second slow roll pa-

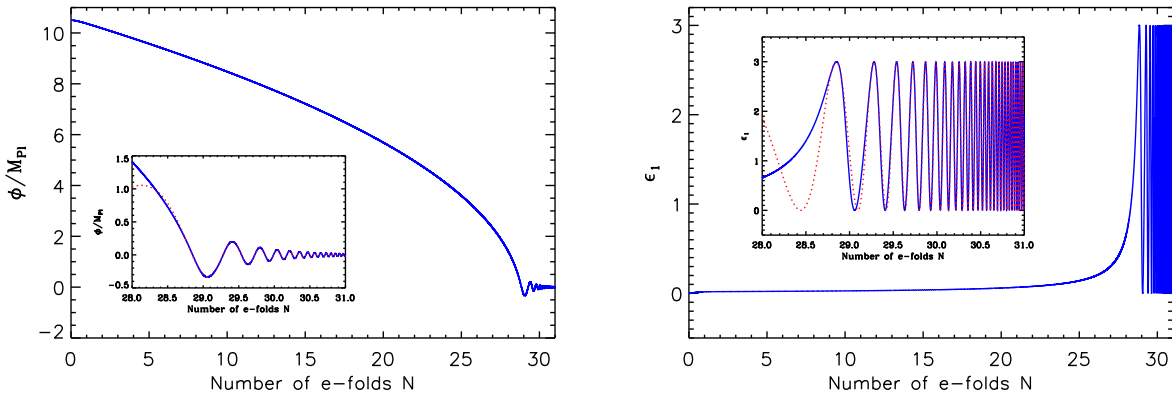


FIG. 1: The behavior of the scalar field (left panel) and the evolution of the first slow roll parameter ϵ_1 (right panel) during the epochs of inflation and preheating have been plotted as a function of the number of e-folds for the case of the archetypical chaotic inflationary model described by the quadratic potential. The blue curves denote the numerical results, while the dotted red curves in the insets represent the analytical results given by Eqs. (4) and (6) that are applicable during preheating. The analytical results evidently match the numerical ones quite well. Note that, for the choice parameters and initial conditions that we have worked with, ϵ_1 turns unity at the e-fold of $N_e \simeq 28.3$, indicating the termination of inflation at the point. The fact that the field oscillates with a smaller and smaller amplitude once inflation has ended is clear from the inset (in the figure on the left panel). We should mention that we have worked with a smaller range of e-folds just for convenience.

parameter ϵ_2 is defined in terms of the first one as follows:

$$\epsilon_2 \equiv \frac{1}{\epsilon_1} \frac{d\epsilon_1}{dN} = \frac{\dot{\epsilon}_1}{H\epsilon_1}. \quad (2)$$

Since the field is oscillating at the bottom of the potential post inflation, $\dot{\phi} = 0$ at the ‘turning points’ and, hence, $\epsilon_1 = 0$ at such instances. Also, when the field is at the bottom of the potential, $V(\phi = 0) = 0$ so that $6H^2 M_{\text{Pl}}^2 = \dot{\phi}^2$, corresponding to $\epsilon_1 = 3$. Hence, we can expect ϵ_1 to oscillate between these two extreme values. Moreover, the above expression for the second slow roll parameter in terms of the first suggests that ϵ_2 will vanish whenever ϵ_1 reaches the maximum value (*i.e.* at the bottom of the potential wherein $\dot{\epsilon}_1 = 0$), and that it will diverge at the ‘turning points’ wherein ϵ_1 vanishes. These behavior too are indeed reflected in the plot of ϵ_1 in Fig. 1 and, in Fig. 2, where we have plotted ϵ_2 .

Let us now try to arrive at the complete behavior of the background scalar field analytically. During the phase of preheating, one finds that the period of the oscillations (characterized by the inverse mass in the case of the quadratic potential of our interest) is much smaller than the time scales associated with the expansion, *i.e.* the inverse of the Hubble parameter H^{-1} . In such a situation, to understand the effects of the scalar field on the scale factor, one can average over the oscillations and make use of the averaged energy density of the scalar field to solve the first Friedmann equation. One finds that, in the quadratic potential of our interest, the expansion behaves as in a matter dominated epoch, with the scale factor growing as $a(t) \propto t^{2/3}$, so that the Hubble parameter behaves as $H = 2/(3t)$ [12, 18, 21, 22].

Therefore, at the time of preheating, the oscillating

scalar field satisfies the differential equation

$$\ddot{\phi} + \frac{2}{t} \dot{\phi} + m^2 \phi = 0. \quad (3)$$

The solution to this differential equation can be immediately written down to be

$$\frac{\phi(t)}{M_{\text{Pl}}} = \frac{\alpha}{mt} \sin(mt + \Delta), \quad (4)$$

where α is a dimensionless constant that we shall soon determine, while Δ is an arbitrary phase chosen suitably to match the transition from inflation to the matter dominated era. The ‘velocity’ of the field is then given by

$$\begin{aligned} \frac{\dot{\phi}(t)}{M_{\text{Pl}}} &= \frac{\alpha}{t} \left[\cos(mt + \Delta) - \frac{1}{mt} \sin(mt + \Delta) \right] \\ &\simeq \frac{\alpha}{t} \cos(mt + \Delta), \end{aligned} \quad (5)$$

where, in arriving at the second expression, for the sake of consistency (*i.e.* in having made use of the averaged energy density in the first Friedmann equation to arrive at the scale factor), we have ignored the second term involving t^{-2} . Upon using the above expressions for ϕ and $\dot{\phi}$ and the fact that $H = 2/(3t)$ in the first Friedmann equation, we obtain that $\alpha = \sqrt{8/3}$. Under these conditions, we find that the first slow roll parameter simplifies to

$$\epsilon_1 \simeq 3 \cos^2(mt + \Delta) \quad (6)$$

which, upon averaging, reduces to 3/2, as is expected in a matter dominated epoch. It is represented in Fig. 1

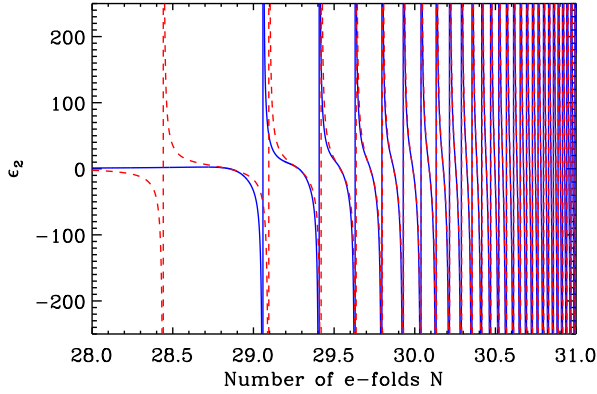


FIG. 2: The behavior of the second slow roll parameter ϵ_2 immediately after the termination of inflation has been plotted as a function of e-folds. As in the previous figure, the blue curve represents the numerical result, while the dashed red curve denotes the analytical result during preheating [viz. Eq. (7)]. Upon comparing this plot with the earlier plot of ϵ_1 , it is clear that ϵ_2 diverges exactly at the turning points where ϵ_1 vanishes, while ϵ_2 itself vanishes whenever the field is at the bottom of the potential at which point ϵ_1 attains its maximum value.

(right panel). On using the above result for ϵ_1 in the definition (2), the second slow roll parameter can be obtained to be

$$\epsilon_2(t) \simeq -3 m t \tan (m t + \Delta), \quad (7)$$

which is illustrated in Fig. 2.

During preheating, we can write $a(t) = a_e (t/t_e)^{2/3}$, where t_e and a_e denote the cosmic time and the scale factor at the end of inflation. We should mention here that, in addition to the phase Δ , one requires the value of t_e in order to match the above analytical results for the scalar field and the slow roll parameters with the numerical results. After setting $t_e = \gamma [2/(3 H_e)]$, where H_e is the value of the Hubble parameter at the end of inflation, we have chosen the quantity γ and the phase Δ suitably so as to match the analytical expressions with the numerical results. It is clear from Figs. 1 and 2 that the agreement between the numerical and the analytical results is indeed very good. We should mention here that, for the results to match, we seem to require a γ that is slightly larger than unity. Actually, for the values of the parameters that we have worked with, we find that we need to choose γ to be about 1.18 for the analytical results to match the numerical ones. The fact that γ is not strictly unity need not come as a surprise. After all, some time is bound to elapse as the universe makes the transition from an inflationary epoch to the behavior as in a matter dominated era.

B. Evolution of the perturbations

As is well known, the scalar perturbations are essentially described by the curvature perturbation, say, \mathcal{R} . The scalar power spectrum $\mathcal{P}_s(k)$ can be defined through the two point correlation function of the Fourier modes of the curvature perturbation as follows [8–11, 16–20]

$$\langle \hat{\mathcal{R}}_{\mathbf{k}} \hat{\mathcal{R}}_{\mathbf{p}} \rangle = \frac{(2\pi)^2}{2k^3} \mathcal{P}_s(k) \delta^{(3)}(\mathbf{k} + \mathbf{p}). \quad (8)$$

The quantum operator $\hat{\mathcal{R}}_{\mathbf{k}}$ in turn is decomposed based on the modes $f_{\mathbf{k}}$ which are governed by the differential equation

$$f_{\mathbf{k}}'' + 2 \frac{z'}{z} f_{\mathbf{k}}' + k^2 f_{\mathbf{k}} = 0, \quad (9)$$

where $z = \sqrt{2\epsilon_1} M_{\text{Pl}} a$. The scalar power spectrum can be expressed in terms of the modes $f_{\mathbf{k}}$ as

$$\mathcal{P}_s(k) \equiv \frac{k^3}{2\pi^2} |f_{\mathbf{k}}|^2 \quad (10)$$

with the assumption that the spectrum is evaluated at suitably late times when the modes are sufficiently outside the Hubble radius during the era of inflation [1–4, 6, 8–11, 16–20].

It often proves to be convenient to work in terms of the so-called Mukhanov-Sasaki variable $v_{\mathbf{k}}$, which is defined as $v_{\mathbf{k}} = z f_{\mathbf{k}}$. In terms of the variable $v_{\mathbf{k}}$, the above equation of motion for $f_{\mathbf{k}}$ reduces to the following simple form:

$$v_{\mathbf{k}}'' + \left(k^2 - \frac{z''}{z} \right) v_{\mathbf{k}} = 0. \quad (11)$$

The initial conditions on the perturbations are imposed when the modes are well inside the Hubble radius during inflation. The modes are usually chosen to be in the Bunch-Davies vacuum, which amounts to demanding that the Mukhanov-Sasaki variable $v_{\mathbf{k}}$ reduces to following Minkowski-like positive frequency mode in the sub-Hubble limit:

$$\lim_{k/(aH) \rightarrow \infty} v_{\mathbf{k}} = \frac{1}{\sqrt{2k}} e^{-i k \eta}. \quad (12)$$

As we shall see, the scalar bi-spectrum shall involve integrals over the modes $f_{\mathbf{k}}$ and its derivative $f_{\mathbf{k}}'$ as well as the slow roll parameters ϵ_1 , ϵ_2 and the derivative ϵ_2' . So, in order to analyze the effects on the bi-spectrum due to preheating, it becomes imperative that, in addition to the behavior of the slow roll parameters, we also understand the evolution of the mode $f_{\mathbf{k}}$ and its derivative during this epoch. We have already studied the behavior of the first two slow roll parameters in the previous sub-section. Therefore, our immediate aim will be to understand the evolution of the curvature perturbations for scales of cosmological interest during the preheating phase.

Since the modes of cosmological interest are well outside the Hubble radius [*i.e.* $k/(aH) \ll 1$] at late times, we need to arrive at the super-Hubble solution for the mode $f_{\mathbf{k}}$ or, equivalently, the Mukhanov-Sasaki variable $v_{\mathbf{k}}$. In a slow roll inflationary regime, *i.e.* when $(\epsilon_1, \epsilon_2, \epsilon_3) \ll 1$, the effective potential z''/z that governs the evolution of $v_{\mathbf{k}}$ [cf. Eq. (11)] can be written as

$$\frac{z''}{z} = a^2 H^2 [2 + \mathcal{O}(\epsilon_1, \epsilon_2, \epsilon_3)] \simeq 2 a^2 H^2. \quad (13)$$

Due to this reason, during slow roll inflation, the super-Hubble condition $k/(aH) \ll 1$ amounts to neglecting the k^2 term with respect to the effective potential z''/z in the differential equation (11). In such a case, it is straightforward to show that the super-Hubble solution to $v_{\mathbf{k}}$ up to the order k^2 can be expressed as follows [4, 6, 8–11, 16–20]:

$$\begin{aligned} v_{\mathbf{k}}(\eta) \simeq & A_{\mathbf{k}} z(\eta) \left[1 - k^2 \int^{\eta} \frac{d\bar{\eta}}{z^2(\bar{\eta})} \int^{\bar{\eta}} d\tilde{\eta} z^2(\tilde{\eta}) \right] \\ & + B_{\mathbf{k}} z(\eta) \int^{\eta} \frac{d\bar{\eta}}{z^2(\bar{\eta})} \\ & \times \left[1 - k^2 \int^{\bar{\eta}} d\tilde{\eta} z^2(\tilde{\eta}) \int^{\tilde{\eta}} \frac{d\check{\eta}}{z^2(\check{\eta})} \right], \end{aligned} \quad (14)$$

where $A_{\mathbf{k}}$ and $B_{\mathbf{k}}$ are k -dependent constants that are determined by the Bunch-Davies initial condition (12) imposed in the sub-Hubble limit. As is well known, the first term involving $A_{\mathbf{k}}$ represents the growing mode, while the second containing $B_{\mathbf{k}}$ corresponds to the decaying mode. However, it is important to realize that, at the time of preheating, the effective potential z''/z is no longer given by the slow roll expression (13). It is clear that the effective potential will contain oscillatory functions and, hence, it can even possibly vanish. So, it is not a priori obvious that one can use the same approach as in the inflationary epoch and simply ignore the k^2 term in the differential equation (11) for arriving at the behavior of the super-Hubble modes. Moreover, it is known that, during the preheating phase, one has to deal with the resonant behavior exhibited by the equation of motion under certain conditions [29, 30, 77]. As a consequence, at this stage, it becomes necessary that we remain cautious and analyze equation (11) more carefully.

In order to study the perturbations during the preheating phase, it proves to be more convenient to work in terms of cosmic time and use a new rescaled variable $\mathcal{V}_{\mathbf{k}}$ that is related to the Mukhanov-Sasaki variable as follows: $\mathcal{V}_{\mathbf{k}} \equiv a^{1/2} v_{\mathbf{k}}$. Then, one finds that Eq. (11) takes the form [29, 30]

$$\begin{aligned} \ddot{\mathcal{V}}_{\mathbf{k}} + & \left[\frac{k^2}{a^2} + \frac{d^2 V}{d\phi^2} + \frac{3\dot{\phi}^2}{M_{\text{Pl}}^2} - \frac{\dot{\phi}^4}{2H^2 M_{\text{Pl}}^4} \right. \\ & \left. + \frac{3}{4M_{\text{Pl}}^2} \left(\frac{\dot{\phi}^2}{2} - V \right) + \frac{2\dot{\phi}}{H M_{\text{Pl}}^2} \frac{dV}{d\phi} \right] \mathcal{V}_{\mathbf{k}} = 0. \end{aligned} \quad (15)$$

Recall that, in the quadratic potential of our interest, soon after inflation, the evolution of the scalar field $\phi(t)$ is given by Eq. (4). Using this solution and its derivative (5), it is then easy to show that, while the third, fourth and the fifth terms within the square brackets in the above differential equation decay as a^{-3} , the last term decays more slowly as it scales as $a^{-3/2}$. Upon retaining only the first, second and the last terms and neglecting the others, one arrives at an equation of the form

$$\begin{aligned} \frac{d^2 \mathcal{V}_{\mathbf{k}}}{d\sigma^2} + & \left[1 + \frac{k^2}{m^2 a^2} \right. \\ & \left. - \frac{4}{m t_e} \left(\frac{a_e}{a} \right)^{3/2} \cos(2\sigma + 2\Delta) \right] \mathcal{V}_{\mathbf{k}} = 0, \end{aligned} \quad (16)$$

where the new independent variable σ is a dimensionless quantity which we have defined to be $\sigma \equiv m t + \pi/4$. We can rewrite the above equation as

$$\frac{d^2 \mathcal{V}_{\mathbf{k}}}{d\sigma^2} + [\mathcal{A}_{\mathbf{k}} - 2q \cos(2\sigma + 2\Delta)] \mathcal{V}_{\mathbf{k}} = 0, \quad (17)$$

with $\mathcal{A}_{\mathbf{k}}$ and q being given by

$$\mathcal{A}_{\mathbf{k}} = 1 + \frac{k^2}{m^2 a^2}, \quad (18)$$

$$q = \frac{2}{m t_e} \left(\frac{a_e}{a} \right)^{3/2}, \quad (19)$$

where, as we mentioned, t_e and a_e denote the cosmic time and the scale factor when inflation ends. The above equation is similar in form to the Mathieu equation (see, for instance, Ref. [78]). The Mathieu equation possesses unstable solutions that are known to grow rapidly when the values of the parameters fall in certain domains known as the resonant bands. As discussed in detail in Refs. [28, 30], since $q \ll 1$ in the situation of our interest, one falls in the narrow resonance regime. In such a case, the first instability band is delineated by the condition $1 - q < \mathcal{A}_{\mathbf{k}} < 1 + q$, which turns out to be equivalent to the condition

$$0 < \frac{k}{a} < \sqrt{3 H m}. \quad (20)$$

It should be emphasized here that the time evolution of the quantities $\mathcal{A}_{\mathbf{k}}$ and q are such that, once a mode has entered the resonance band, it remains inside it during the entire oscillatory phase.

Note that, in Eq. (16), we can neglect the term involving k^2 provided $k^2/(m^2 a^2) \ll 1$. This condition can be rewritten as

$$\left(\frac{k}{a H} \right)^2 \frac{H^2}{m^2} \ll 1. \quad (21)$$

On the other hand, the condition to fall in the first instability band, viz. Eq. (20), can be expressed as [30]

$$\left(\frac{k}{a H} \right)^2 \frac{H}{3 m} \ll 1. \quad (22)$$

Given that, $H < m$ immediately after inflation, it is evident that the first of the above two conditions will be satisfied if the second is. In other words, being in the first instability band implies that one can indeed neglect the k^2 term in Eq. (16). But, clearly, this is completely equivalent to ignoring the k^2 term in the original equation (11). Therefore, we can conclude that, provided we fall in the first instability band (which is the case for the range of modes and parameters of our interest), it is perfectly valid to work with the super-Hubble solution (14) even during the preheating phase.

The above conclusion can also be supported by the following arguments. As discussed in Ref. [30], in the first instability band, the Floquet index is given by $\mu = q/2$. In such a case, the mode $\mathcal{V}_{\mathbf{k}}$ behaves as $\mathcal{V}_{\mathbf{k}} \propto e^{\mu\sigma}$. However, in the situation of our interest, since we have a time dependent Floquet index, the corresponding solution can be written as

$$\mathcal{V}_{\mathbf{k}} \propto \exp\left(\int \mu d\sigma\right) \propto a^{3/2} \quad (23)$$

which, in turn, implies that $v_{\mathbf{k}} = \mathcal{V}_{\mathbf{k}}/a^{1/2} \propto a$. Further, since, $f_{\mathbf{k}} = v_{\mathbf{k}}/z$ and $z \propto a$ during preheating (i.e. if one makes use of the fact that $\epsilon_1 = 3/2$ on the average), we arrive at the result that $f_{\mathbf{k}}$ remains a constant during this phase. This property is indeed the well known behavior one obtains if one simply retains the very first term of the growing mode in the super-Hubble solution (14). But, it should be realized that the above arguments also demonstrate another important point. Note that $f_{\mathbf{k}}$ is a constant not only for modes on super-Hubble scales but, for all the modes (even those that remain in the sub-Hubble domain), provided they fall in the resonance band. This behavior can possibly be attributed to the background. After all, it is common knowledge that the amplitude of the curvature perturbations remain constant on all scales in a matter dominated era [1–4, 6, 8–11, 16–20].

We have proven that, in the first instability band and on super-Hubble scales, the solution (14) is valid during preheating. Let us now analyze this solution in further detail. In particular, in order to check the extent of its validity, let us compare the analytical estimate for the curvature perturbation with the numerical solution. It is

clear that the solution (14) leads to the following expression for the growing mode:

$$f_{\mathbf{k}}(\eta) \simeq A_{\mathbf{k}} \left[1 - k^2 \int^{\eta} \frac{d\bar{\eta}}{z^2(\bar{\eta})} \int^{\bar{\eta}} d\tilde{\eta} z^2(\tilde{\eta}) \right], \quad (24)$$

where we have retained the scale dependent correction for comparison with the numerical result. Since it is the contribution due to the growing mode that will prove to be dominant, we shall compare the behavior of the above solution during preheating with the corresponding numerical result. We shall carry out the comparison for a suitably small scale mode so that the second term in the above expression is not completely insignificant.

We now need to evaluate the double integral in the above expression for $f_{\mathbf{k}}$ during preheating. Let us write

$$K(t) = \int^{\eta} \frac{d\bar{\eta}}{z^2(\bar{\eta})} J(\bar{\eta}) = \int^t \frac{d\bar{t}}{a(\bar{t}) z^2(\bar{t})} J(\bar{t}), \quad (25)$$

where

$$J(\bar{t}) = \int^{\bar{\eta}} d\tilde{\eta} z^2(\tilde{\eta}) = \int^{\bar{t}} \frac{d\tilde{t}}{a(\tilde{t})} z^2(\tilde{t}). \quad (26)$$

Upon making use of the matter dominated behavior of the scale factor and the expression (6) for ϵ_1 , we find that the integral $J(\bar{t})$ can be performed exactly. We obtain that

$$\begin{aligned} J(\bar{t}) = & \frac{3}{2} M_{\text{Pl}}^2 a_e t_e \left[\frac{6}{5} \left(\frac{\bar{t}}{t_e} \right)^{5/3} \right. \\ & + e^{2i\Delta} (-2im t_e)^{-5/3} \gamma\left(\frac{5}{3}, -2im\bar{t}\right) \\ & \left. + e^{-2i\Delta} (2im t_e)^{-5/3} \gamma\left(\frac{5}{3}, 2im\bar{t}\right) + \mathcal{C} \right], \quad (27) \end{aligned}$$

where $\gamma(b, x)$ is the incomplete Gamma function (see, for example, Refs. [79, 80]), while the quantity \mathcal{C} is a dimensionless constant of integration. Then, the relation between the incomplete Gamma and the Gamma functions allows us to express the function $K(t)$ as follows:

$$\begin{aligned} K(t) \simeq & \frac{t_e}{4a_e^2} \int^t \frac{d\bar{t}}{\cos^2(m\bar{t} + \Delta)} \left[\frac{6}{5} \left(\frac{t_e}{\bar{t}} \right)^{1/3} + e^{2i\Delta} (-2im t_e)^{-5/3} \Gamma\left(\frac{5}{3}\right) \left(\frac{t_e}{\bar{t}} \right)^2 - e^{2i(m\bar{t} + \Delta)} (-2im t_e)^{-1} \left(\frac{t_e}{\bar{t}} \right)^{4/3} \right. \\ & \left. + e^{-2i\Delta} (2im t_e)^{-5/3} \Gamma\left(\frac{5}{3}\right) \left(\frac{t_e}{\bar{t}} \right)^2 - e^{-2i(m\bar{t} + \Delta)} (2im t_e)^{-1} \left(\frac{t_e}{\bar{t}} \right)^{4/3} + \mathcal{C} \left(\frac{t_e}{\bar{t}} \right)^2 \right] \\ \simeq & \frac{3t_e^{4/3}}{10a_e^2} \int^t \frac{d\bar{t} \bar{t}^{-1/3}}{\cos^2(m\bar{t} + \Delta)} + \dots \end{aligned} \quad (28)$$

In arriving at the final equality, we have used the asymp-

totic property of the incomplete Gamma function [79, 80]

and have retained only the dominant term in inverse power of mt . The final expression above can be integrated by parts to arrive at

$$K(t) \simeq \frac{3t_e^{4/3}}{10ma_e^2} t^{-1/3} \tan(mt + \Delta) + \frac{t_e^{4/3}}{10ma_e^2} \int^t d\bar{t} \bar{t}^{-4/3} \tan(m\bar{t} + \Delta). \quad (29)$$

The second term containing the integral in this expression is of the order of the other terms that we have already neglected and, hence, it too can be ignored. As a result, the growing mode of the curvature perturbation can be written as

$$f_k \simeq A_k \left[1 - \frac{3}{10} \frac{k^2 t_e^{4/3}}{a_e^2 m t^{1/3}} \tan(mt + \Delta) \right] = A_k \left[1 - \frac{1}{5} \left(\frac{k}{aH} \right)^2 \frac{H}{m} \tan(mt + \Delta) \right], \quad (30)$$

in perfect agreement with the result that has been obtained recently in the literature [32]. It is evident from the above expression that the evolution of the curvature perturbation will contain sharp spikes during preheating, a feature that is clearly visible in Fig. 3 wherein we have plotted the above analytical expression as well as the corresponding numerical result (in this context, also see Fig. 4 in Ref. [30] where the spikes are also clearly visible).

It is important that we make a couple of remarks concerning the appearance of the spikes in the evolution of the curvature perturbation. Firstly, as the spikes are encountered both analytically and numerically, evidently, they are not artifacts of the adopted approach. Secondly, one may fear that the perturbation theory would break down as soon as one encounters a spike, which indicates a rather large value for the perturbation variable of interest. We believe that such issues could possibly be avoided when one couples the inflaton to radiation, as is needed to reheat the universe.

III. THE SCALAR BI-SPECTRUM IN THE MALDACENA FORMALISM

Now that we have understood the behavior of the large scale modes at the time of preheating, let us turn to in-

vestigate the effects of preheating on the bi-spectrum. In this section, we shall quickly sketch the various contributions to the bi-spectrum in the Maldacena formalism [37].

The scalar bi-spectrum $\mathcal{B}_s(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ is defined in terms of the three point correlation function of the Fourier modes of the curvature perturbation \mathcal{R} as follows [75, 76]:

$$\langle \hat{\mathcal{R}}_{\mathbf{k}_1} \hat{\mathcal{R}}_{\mathbf{k}_2} \hat{\mathcal{R}}_{\mathbf{k}_3} \rangle = (2\pi)^3 \mathcal{B}_s(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \times \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3). \quad (31)$$

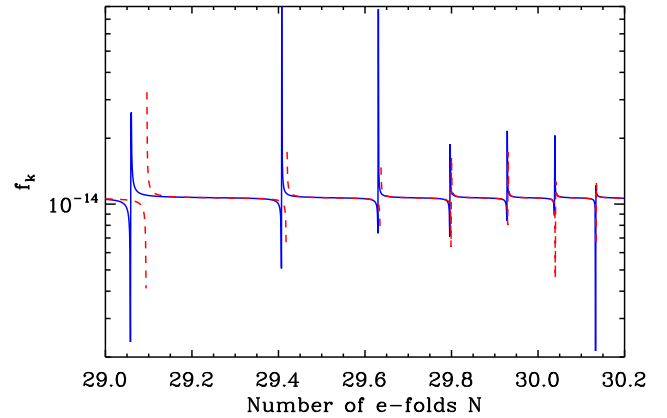


FIG. 3: The behavior of the curvature perturbation during preheating. The blue curve denotes the numerical result, while the dashed red curve represents the analytical solution (30). We have chosen to work with a very small scale mode k that leaves the Hubble radius at about two e-folds before the end of inflation. We have made use of the same value of Δ as in the previous two figures and we have fixed A_k [cf. Eq. (30)] by choosing it to be the numerical value of the curvature perturbation at a suitable time close to the end of inflation. It is clear that the agreement between the analytical and the numerical results is quite good.

For convenience, we shall set

$$G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = (2\pi)^{9/2} \mathcal{B}_s(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3). \quad (32)$$

In the Maldacena formalism to calculate the bi-spectrum [37], the quantity $G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ can be expressed as [38, 39, 43, 51, 52]

$$\begin{aligned}
G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) &\equiv \sum_{C=1}^7 G_C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \\
&\equiv M_{\text{Pl}}^2 \sum_{C=1}^6 \left[f_{\mathbf{k}_1}(\eta_f) f_{\mathbf{k}_2}(\eta_f) f_{\mathbf{k}_3}(\eta_f) \mathcal{G}_C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) + f_{\mathbf{k}_1}^*(\eta_f) f_{\mathbf{k}_2}^*(\eta_f) f_{\mathbf{k}_3}^*(\eta_f) \mathcal{G}_C^*(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \right] \\
&\quad + G_7(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3),
\end{aligned} \tag{33}$$

where η_f denotes the final time when the bi-spectrum is to be evaluated. The quantities $\mathcal{G}_C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ with $C = (1, 6)$ are described by the integrals [38, 39, 43, 51, 52]

$$\mathcal{G}_1(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2i \int_{\eta_i}^{\eta_f} d\eta a^2 \epsilon_1^2 (f_{\mathbf{k}_1}^* f_{\mathbf{k}_2}' f_{\mathbf{k}_3}' + \text{two permutations}), \tag{34}$$

$$\mathcal{G}_2(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = -2i (\mathbf{k}_1 \cdot \mathbf{k}_2 + \text{two permutations}) \int_{\eta_i}^{\eta_f} d\eta a^2 \epsilon_1^2 f_{\mathbf{k}_1}^* f_{\mathbf{k}_2}^* f_{\mathbf{k}_3}', \tag{35}$$

$$\mathcal{G}_3(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = -2i \int_{\eta_i}^{\eta_f} d\eta a^2 \epsilon_1^2 \left[\left(\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_2^2} \right) f_{\mathbf{k}_1}^* f_{\mathbf{k}_2}' f_{\mathbf{k}_3}' + \text{five permutations} \right], \tag{36}$$

$$\mathcal{G}_4(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = i \int_{\eta_i}^{\eta_f} d\eta a^2 \epsilon_1 \epsilon_2' (f_{\mathbf{k}_1}^* f_{\mathbf{k}_2}^* f_{\mathbf{k}_3}' + \text{two permutations}), \tag{37}$$

$$\mathcal{G}_5(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{i}{2} \int_{\eta_i}^{\eta_f} d\eta a^2 \epsilon_1^3 \left[\left(\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_2^2} \right) f_{\mathbf{k}_1}^* f_{\mathbf{k}_2}' f_{\mathbf{k}_3}' + \text{five permutations} \right], \tag{38}$$

$$\mathcal{G}_6(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{i}{2} \int_{\eta_i}^{\eta_f} d\eta a^2 \epsilon_1^3 \left\{ \left[\frac{k_1^2 (\mathbf{k}_2 \cdot \mathbf{k}_3)}{k_2^2 k_3^2} \right] f_{\mathbf{k}_1}^* f_{\mathbf{k}_2}' f_{\mathbf{k}_3}' + \text{two permutations} \right\}, \tag{39}$$

where η_i denotes the time when the modes $f_{\mathbf{k}}$ are well inside the Hubble radius during inflation. The additional, seventh term $G_7(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ arises due to a field redefinition, and its contribution to $G(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ is found to be

$$\begin{aligned}
G_7(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) &= \frac{\epsilon_2(\eta_f)}{2} \left[|f_{\mathbf{k}_2}(\eta_f)|^2 |f_{\mathbf{k}_3}(\eta_f)|^2 \right. \\
&\quad \left. + \text{two permutations} \right].
\end{aligned} \tag{40}$$

IV. THE CONTRIBUTIONS TO THE SCALAR BI-SPECTRUM DURING PREHEATING

Our goal now is to determine the different contributions to the bi-spectrum due to the epoch of preheating for modes of cosmological interest. Since we know the behavior of the slow roll parameters and the large scale modes during preheating, it is clear from the above expressions that, it is simply a matter of being able to evaluate the integrals involved. As we shall illustrate, it turns out to be possible to actually evaluate these integrals explicitly and thereby arrive at the contributions to the bi-spectrum due to preheating.

During preheating, for the large scale modes (i.e. for $k \ll aH$), the contribution to $f_{\mathbf{k}}$ is dominated by the

constant growing mode [viz. the very first term in Eq. (14)] so that one has

$$f_{\mathbf{k}} \simeq A_{\mathbf{k}}. \tag{41}$$

As the corresponding derivative trivially vanishes, at the same order in k , the leading non-zero contribution to the quantity $f_{\mathbf{k}}'$ is determined by the decaying mode, which is given by

$$f_{\mathbf{k}} \simeq B_{\mathbf{k}} \int^{\eta} \frac{d\bar{\eta}}{z^2(\bar{\eta})} \tag{42}$$

and, hence,

$$f_{\mathbf{k}}' \simeq \frac{B_{\mathbf{k}}}{z^2} = \frac{\bar{B}_{\mathbf{k}}}{a^2 \epsilon_1}, \tag{43}$$

where we have set $\bar{B}_{\mathbf{k}} = B_{\mathbf{k}}/(2M_{\text{Pl}}^2)$.

Let us now turn to discuss the various contributions to the bi-spectrum as the background and the modes evolve during preheating.

A. The fourth and the seventh terms

As we had mentioned in the introductory section, it has been realized that departures from slow roll during

inflation can lead to significant non-Gaussianities [44–52]. In such situations, it has been repeatedly noticed that it is the fourth term, viz. $G_4(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$, that contributes the most to the bi-spectrum because it involves the derivative of the second slow roll parameter ϵ_2 . Since ϵ_2 grows extremely large during preheating [see Eq. (7) as well as Fig. 2], it is natural to expect that the fourth term will contribute significantly to the bi-spectrum at the time of preheating. So, we shall first investigate the contribution due to $G_4(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$. As we shall see, it indeed proves to be large during preheating. However, interestingly, as we shall illustrate, this large contribution is canceled by a similar contribution due to the seventh term $G_7(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ which also involves the second slow roll parameter ϵ_2 .

Upon using the super-Hubble behavior (41) and (43) of the mode $f_{\mathbf{k}}$ and its derivative in the expression (37), we obtain that

$$\mathcal{G}_4(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \simeq i (A_{\mathbf{k}_1}^* A_{\mathbf{k}_2}^* \bar{B}_{\mathbf{k}_3}^* + \text{two permutations}) \times \int_{\eta_e}^{\eta_f} d\eta \epsilon_2', \quad (44)$$

where η_e denotes the time at which inflation ends. The above expression can be trivially integrated to yield

$$\mathcal{G}_4(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \simeq i (A_{\mathbf{k}_1}^* A_{\mathbf{k}_2}^* \bar{B}_{\mathbf{k}_3}^* + \text{two permutations}) \times [\epsilon_2(\eta_f) - \epsilon_2(\eta_e)], \quad (45)$$

so that the corresponding contribution to the bi-spectrum can be expressed as

$$G_4(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \simeq i M_{\text{Pl}}^2 [\epsilon_2(\eta_f) - \epsilon_2(\eta_e)] \times \left[|A_{\mathbf{k}_1}|^2 |A_{\mathbf{k}_2}|^2 (A_{\mathbf{k}_3} \bar{B}_{\mathbf{k}_3}^* - A_{\mathbf{k}_3}^* \bar{B}_{\mathbf{k}_3}) + \text{two permutations} \right]. \quad (46)$$

Since this expression is proportional to ϵ_2 , it suggests that preheating may contribute substantially to the bi-spectrum. But, as we shall soon show, this large contribution is canceled by a similar contribution from the seventh term that arises due to the field redefinition [cf. Eq. (40)].

Now, consider the Wronskian

$$\mathcal{W} = f_{\mathbf{k}} f_{\mathbf{k}}'^* - f_{\mathbf{k}}^* f_{\mathbf{k}}'. \quad (47)$$

Upon using the equation of motion (9) for $f_{\mathbf{k}}$, one can show that, $\mathcal{W} = W/z^2$, where W is a constant. It is important to note that this result is valid on all scales, even in the sub-Hubble limit during inflation. In this limit, as we had mentioned, the modes $v_{\mathbf{k}}$ satisfy the Bunch-Davies initial condition (12). On making use of this sub-Hubble behavior in the above definition of the Wronskian \mathcal{W} , one obtains that $W = i$. In the super-Hubble limit, we have, on using the corresponding solution (41) and its derivative (43),

$$\mathcal{W} = \frac{2 M_{\text{Pl}}^2}{z^2} (A_{\mathbf{k}} \bar{B}_{\mathbf{k}}^* - A_{\mathbf{k}}^* \bar{B}_{\mathbf{k}}) = \frac{i}{z^2}. \quad (48)$$

Therefore, we obtain that

$$(A_{\mathbf{k}} \bar{B}_{\mathbf{k}}^* - A_{\mathbf{k}}^* \bar{B}_{\mathbf{k}}) = \frac{i}{2 M_{\text{Pl}}^2}, \quad (49)$$

and, hence, the expression (46) for $G_4(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ simplifies to

$$G_4(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \simeq -\frac{1}{2} [\epsilon_2(\eta_f) - \epsilon_2(\eta_e)] \times \left[|A_{\mathbf{k}_1}|^2 |A_{\mathbf{k}_2}|^2 + \text{two permutations} \right]. \quad (50)$$

Note that the first of these terms [involving $\epsilon_2(\eta_f)$] *exactly* cancels the contribution $G_7(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ [cf. Eq. (40)] that arises due to the field redefinition (with $f_{\mathbf{k}}$ set to $A_{\mathbf{k}}$), leaving behind only the contributions generated during inflation.

Before we go on to discuss the behavior of the other contributions, we should emphasize here that the above result for the fourth and the seventh terms applies to all single field models. It is important to appreciate the fact that we have made no assumptions whatsoever about the inflationary potential in arriving at the above conclusion. However, one should keep in mind that, regarding its behavior near the minima, we have made use of the fact that the potential can be approximated by a parabola. Indeed, it is with this explicit form that we have been able to identify a solution to the Mukhanov-Sasaki equation that leads to a constant curvature perturbation.

B. The second term

We shall now turn to the second term $G_2(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$, which in certain situations is known to be as large as the fourth term when there exist periods of fast roll [51, 52]. During preheating, we have

$$G_2(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = -2i (\mathbf{k}_1 \cdot \mathbf{k}_2 + \text{two permutations}) \times A_{\mathbf{k}_1}^* A_{\mathbf{k}_2}^* A_{\mathbf{k}_3}^* I_2(\eta_f, \eta_e), \quad (51)$$

where $I_2(\eta_f, \eta_e)$ denotes the integral

$$I_2(\eta_f, \eta_e) = \int_{\eta_e}^{\eta_f} d\eta a^2 \epsilon_1^2, \quad (52)$$

so that the corresponding contribution to the bi-spectrum is given by

$$G_2(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = -2i M_{\text{Pl}}^2 (\mathbf{k}_1 \cdot \mathbf{k}_2 + \text{two permutations}) \times |A_{\mathbf{k}_1}|^2 |A_{\mathbf{k}_2}|^2 |A_{\mathbf{k}_3}|^2 \times [I_2(\eta_f, \eta_e) - I_2^*(\eta_f, \eta_e)], \quad (53)$$

which identically vanishes since I_2 is real. Needless to add, this implies that the second term does not contribute to the bi-spectrum during preheating. Again we

should emphasize the fact that, as in the case of the fourth and the seventh terms, this result holds good for any inflationary model provided it can be approximated by a parabola in the vicinity of its minimum.

In order to check that our assumptions and approximations are indeed valid, let us now estimate the quantity $\mathcal{G}_2(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ analytically during preheating for the case

$$\begin{aligned}
I_2(\eta_f, \eta_e) = & \frac{9 a_e t_e}{16} (m t_e)^{-5/3} \left(\frac{18}{5} (m t_e)^{5/3} \left[e^{5(N_f - N_e)/2} - 1 \right] \right. \\
& + 4 (-2i)^{-5/3} e^{2i\Delta} \left\{ \gamma \left[\frac{5}{3}, -2i m t_e e^{3(N_f - N_e)/2} \right] - \gamma \left(\frac{5}{3}, -2i m t_e \right) \right\} \\
& + 4 (2i)^{-5/3} e^{-2i\Delta} \left\{ \gamma \left[\frac{5}{3}, 2i m t_e e^{3(N_f - N_e)/2} \right] - \gamma \left(\frac{5}{3}, 2i m t_e \right) \right\} \\
& + (-4i)^{-5/3} e^{4i\Delta} \left\{ \gamma \left[\frac{5}{3}, -4i m t_e e^{3(N_f - N_e)/2} \right] - \gamma \left(\frac{5}{3}, -4i m t_e \right) \right\} \\
& \left. + (4i)^{-5/3} e^{-4i\Delta} \left\{ \gamma \left[\frac{5}{3}, 4i m t_e e^{3(N_f - N_e)/2} \right] - \gamma \left(\frac{5}{3}, 4i m t_e \right) \right\} \right). \quad (54)
\end{aligned}$$

On the other hand, had we ignored the oscillations during preheating, and assumed that the background behavior is exactly the same as in a matter dominated era, then, since, $\langle \epsilon_1 \rangle = 3/2$, the quantity $\langle I_2(\eta_f, \eta_e) \rangle$ can be trivially evaluated to yield

$$\langle I_2(\eta_f, \eta_e) \rangle = \frac{27 a_e t_e}{20} \left[e^{5(N_f - N_e)/2} - 1 \right]. \quad (55)$$

We have plotted the quantity $\mathcal{G}_2(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ in the equilateral limit, *i.e.* when $k_1 = k_2 = k_3 = k$, corresponding to the analytical expressions (54) and (55) as well as the numerical result as a function of upper limit N_f during preheating in Fig. 4. The agreement between the analytical and the numerical results is indeed striking.

C. The remaining terms

In this sub-section, we shall analytically compute the contributions due to the remaining terms, *viz.* the first, third, fifth and the sixth. Notice that, the first term $G_1(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ and the third term $G_3(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ involve the same integrals. Therefore, these two contributions to the bi-spectrum can be clubbed together. Similarly, the fifth and the sixth terms, *viz.* $G_5(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ and $G_6(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$, also contain integrals of the same type, and hence their contributions too can be combined.

On making use of the super-Hubble behavior (41) and (43) of the mode $f_{\mathbf{k}}$ and its derivative, we obtain

of the quadratic potential and compare with the corresponding numerical result. In such a case, the integral $I_2(\eta_f, \eta_e)$ can be carried out along similar lines to the integral $J(\bar{t})$ that we had evaluated earlier [cf. Eq. (27)]. We find that it can be expressed in terms of the incomplete Gamma function $\gamma(b, x)$ as follows:

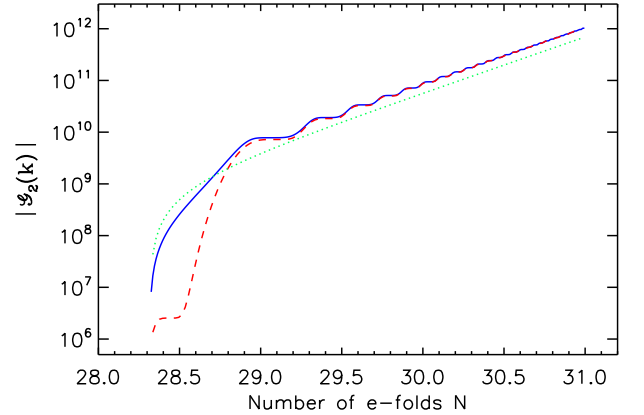


FIG. 4: The behavior of the quantity $\mathcal{G}_2(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ in the equilateral limit, *i.e.* when $k_1 = k_2 = k_3 = k$, for a mode that leaves the Hubble radius at about 20 e-folds before the end of inflation. The blue curve represents the numerical result. The dashed red curve denotes the analytical result arrived at using the integral (54) and with the same choice of Δ as in the earlier figures. The dotted green curve corresponds to the integral (55) obtained when the oscillations have been ignored. As in the previous figure, the value of $A_{\mathbf{k}}$ has been fixed by choosing it to be the numerical value of the curvature perturbation on super-Hubble scales. Needless to add, the match between the analytical and the numerical results is excellent.

that, during preheating,

$$\mathcal{G}_1(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \simeq 2i (A_{\mathbf{k}_1}^* \bar{B}_{\mathbf{k}_2}^* \bar{B}_{\mathbf{k}_3}^* + \text{two permutations}) \times I_{13}(\eta_f, \eta_e) \quad (56)$$

and

$$\mathcal{G}_3(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \simeq -2i \left[\left(\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_2^2} \right) A_{\mathbf{k}_1}^* \bar{B}_{\mathbf{k}_2}^* \bar{B}_{\mathbf{k}_3}^* + \text{five permutations} \right] I_{13}(\eta_f, \eta_e). \quad (57)$$

The quantity $I_{13}(\eta_f, \eta_e)$ represents the integral

$$I_{13}(\eta_f, \eta_e) = \int_{\eta_e}^{\eta_f} \frac{d\eta}{a^2}, \quad (58)$$

which can be trivially carried out during preheating to yield

$$I_{13}(\eta_f, \eta_e) = \frac{t_e}{a_e^3} \left[1 - e^{-3(N_f - N_e)/2} \right]. \quad (59)$$

From these results, we find that the contribution to the bi-spectrum due to the first and the third terms can be written as

$$G_1(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) + G_3(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2i M_{\text{Pl}}^2 I_{13}(\eta_f, \eta_e) \left[\left(1 - \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_2^2} - \frac{\mathbf{k}_1 \cdot \mathbf{k}_3}{k_3^2} \right) |A_{\mathbf{k}_1}|^2 \times (A_{\mathbf{k}_2} \bar{B}_{\mathbf{k}_2}^* A_{\mathbf{k}_3} \bar{B}_{\mathbf{k}_3}^* - A_{\mathbf{k}_2}^* \bar{B}_{\mathbf{k}_2} A_{\mathbf{k}_3}^* \bar{B}_{\mathbf{k}_3}) + \text{two permutations} \right]. \quad (60)$$

Since the second term in the above expression for $I_{13}(\eta_f, \eta_e)$ dies quickly with growing N_f , the corresponding contribution to the bi-spectrum proves to be negligible.

The contributions due to the fifth and the sixth terms during preheating can be arrived at in a similar fashion. We obtain that

$$G_5(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) + G_6(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{i M_{\text{Pl}}^2}{2} I_{56}(\eta_f, \eta_e) \left\{ \left[\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_2^2} + \frac{\mathbf{k}_1 \cdot \mathbf{k}_3}{k_3^2} + \frac{k_1^2 (\mathbf{k}_2 \cdot \mathbf{k}_3)}{k_2^2 k_3^2} \right] |A_{\mathbf{k}_1}|^2 (A_{\mathbf{k}_2} \bar{B}_{\mathbf{k}_2}^* A_{\mathbf{k}_3} \bar{B}_{\mathbf{k}_3}^* - A_{\mathbf{k}_2}^* \bar{B}_{\mathbf{k}_2} A_{\mathbf{k}_3}^* \bar{B}_{\mathbf{k}_3}) + \text{two permutations} \right\}, \quad (61)$$

with $I_{56}(\eta_f, \eta_e)$ denoting the integral

$$I_{56}(\eta_f, \eta_e) = \int_{\eta_e}^{\eta_f} \frac{d\eta}{a^2} \epsilon_1. \quad (62)$$

This integral too can be evaluated rather easily to arrive at the following expression:

$$I_{56}(\eta_f, \eta_e) = \frac{3t_e}{a_e^3} \left(\cos^2(m t_e + \Delta) - e^{-3(N_f - N_e)/2} \cos^2[m t_e e^{3(N_f - N_e)/2} + \Delta] \right. \\ \left. + m t_e \cos(2\Delta) \left\{ \text{Si}(2m t_e) - \text{Si} \left[2m t_e e^{3(N_f - N_e)/2} \right] \right\} \right. \\ \left. + m t_e \sin(2\Delta) \left\{ \text{Ci}(2m t_e) - \text{Ci} \left[2m t_e e^{3(N_f - N_e)/2} \right] \right\} \right), \quad (63)$$

where $\text{Si}(x)$ and $\text{Ci}(x)$ are the sine and the cosine integral functions [79, 80]. And, had we ignored the oscillations,

we would have arrived at

$$\langle\langle I_{56}(\eta_f, \eta_e) \rangle\rangle = \frac{3t_e}{2a_e^3} \left[1 - e^{-3(N_f - N_e)/2} \right], \quad (64)$$

which is of the same order as $I_{13}(\eta_f, \eta_e)$, and hence completely negligible as we had discussed.

D. The contribution to f_{NL} during preheating

Let us now actually estimate the extent of the contribution to the non-Gaussianity parameter f_{NL} during preheating. Since the contributions due to the combination of the fourth plus the seventh and the second term completely vanish at late times, the non-zero contribution to the bi-spectrum during preheating is determined by the first, third, fifth and the sixth terms. Note that, if one ignores the oscillations post-inflation, then one has $I_{56}(\eta_f, \eta_e) = 3 I_{13}(\eta_f, \eta_e)/2$. In such a situation, we find that the non-trivial contributions lead to the following bi-spectrum in the simpler case of the equilateral limit:

$$G_{\text{eq}}(k) = \frac{69 i M_{\text{Pl}}^2}{8} I_{13}(\eta_f, \eta_e) |A_{\mathbf{k}}|^2 \times (A_{\mathbf{k}}^2 \bar{B}_{\mathbf{k}}^{*2} - A_{\mathbf{k}}^{*2} \bar{B}_{\mathbf{k}}^2). \quad (65)$$

In the equilateral limit, the non-Gaussianity parameter f_{NL} is given by

$$f_{\text{NL}}^{\text{eq}}(k) = -\frac{10}{9} \frac{1}{(2\pi)^4} \frac{k^6 G_{\text{eq}}(k)}{\mathcal{P}_s^2(k)}, \quad (66)$$

where $\mathcal{P}_s(k)$ is the power spectrum defined in Eq. (8). Upon making use of the fact that $f_{\mathbf{k}} \simeq A_{\mathbf{k}}$ at late times, we then obtain the contribution to $f_{\text{NL}}^{\text{eq}}$ during preheating to be

$$f_{\text{NL}}^{\text{eq}}(k) \simeq -\frac{115 i M_{\text{Pl}}^2}{48} I_{13}(\eta_f, \eta_e) \times \left(\frac{A_{\mathbf{k}}^2 \bar{B}_{\mathbf{k}}^{*2} - A_{\mathbf{k}}^{*2} \bar{B}_{\mathbf{k}}^2}{|A_{\mathbf{k}}|^2} \right). \quad (67)$$

In order to explicitly calculate the parameter f_{NL} , we need to first specify the inflationary scenario. We shall choose to work with power law inflation because it permits exact calculations, and it can also mimic slow roll inflation. During power law inflation, the scale factor can be written as $a(\eta) = a_1 (\eta/\eta_1)^{\beta+1}$, where a_1 and η_1 are constants, while β is a free index. It is useful to note that,

in such a case, the first slow roll parameter is a constant and is given by $\epsilon_1 = (\beta + 2)/(\beta + 1)$. The current constraints on the scalar spectral index suggest that $\beta \lesssim -2$, which implies that the corresponding scale factor is close to that of de Sitter.

In power law inflation, the exact solution to Eq. (11) can be expressed in terms of the Bessel function $J_\nu(x)$ as follows:

$$v_{\mathbf{k}}(\eta) = \sqrt{-k\eta} [C_{\mathbf{k}} J_\nu(-k\eta) + D_{\mathbf{k}} J_{-\nu}(-k\eta)], \quad (68)$$

where $\nu = (\beta + 1/2)$, and the quantities $C_{\mathbf{k}}$ and $D_{\mathbf{k}}$ are constants that are determined by the initial conditions. Upon demanding that the above solution satisfies the Bunch-Davies initial condition (12), one obtains that

$$C_{\mathbf{k}} = -D_{\mathbf{k}} e^{-i\pi(\beta+1/2)}, \quad (69)$$

$$D_{\mathbf{k}} = \sqrt{\frac{\pi}{k}} \frac{e^{i\pi\beta/2}}{2 \cos(\pi\beta)}. \quad (70)$$

One can confirm that the super-Hubble limit of the solution (68) above indeed reproduces Eq. (14) exactly. Moreover, the limit also allows us to arrive at the constants $A_{\mathbf{k}}$ and $B_{\mathbf{k}}$, which are found to be

$$A_{\mathbf{k}} = \frac{2^{-(\beta+1/2)}}{\Gamma(\beta+3/2)} \frac{(-k\eta_1)^{\beta+1}}{\sqrt{2}\epsilon_1 a_1 M_{\text{Pl}}} C_{\mathbf{k}}, \quad (71)$$

$$B_{\mathbf{k}} = -\frac{(2\beta+1) 2^{\beta+1/2}}{\Gamma(-\beta+1/2)} \frac{\sqrt{2}\epsilon_1 a_1 M_{\text{Pl}}}{\eta_1} \times (-k\eta_1)^{-\beta} D_{\mathbf{k}}. \quad (72)$$

Then, upon inserting the above expressions for the quantities $A_{\mathbf{k}}$ and $B_{\mathbf{k}}$ in Eq. (67), we find that

$$f_{\text{NL}}^{\text{eq}}(k) = \frac{115\epsilon_1}{288\pi} \Gamma^2\left(\beta + \frac{1}{2}\right) 2^{2\beta+1} (2\beta+1)^2 \times \sin(2\pi\beta) |\beta+1|^{-2(\beta+1)} \times \left[1 - e^{-3(N_f - N_e)/2}\right] \left(\frac{k}{a_e H_e}\right)^{-(2\beta+1)}. \quad (73)$$

This expression can also be rewritten in terms of the parameters describing the post-inflationary evolution. We obtain that

$$f_{\text{NL}}^{\text{eq}}(k) = \frac{115\epsilon_1}{288\pi} \Gamma^2\left(\beta + \frac{1}{2}\right) 2^{2\beta+1} (2\beta+1)^2 \sin(2\pi\beta) |\beta+1|^{-2(\beta+1)} \left[1 - e^{-3(N_f - N_e)/2}\right] \times \left[\left(\frac{\pi^2 g_*}{30}\right)^{-1/4} (1+z_{\text{eq}})^{1/4} \frac{\rho_{\text{cri}}^{1/4}}{T_{\text{rh}}}\right]^{-(2\beta+1)} \left(\frac{k}{a_0 H_0}\right)^{-(2\beta+1)}, \quad (74)$$

where g_* denotes the effective number of relativistic de-

grees of freedom at reheating, T_{rh} the reheating tem-

perature and z_{eq} the redshift at the epoch of equality. Also, ρ_{cri} , a_0 and H_0 represent the critical energy density, the scale factor and the Hubble parameter today, respectively. The above expression is mainly determined by the ratio $\rho_{\text{cri}}^{1/4}/T_{\text{rh}}$. For a model with $\beta \simeq -2$ and a reheating temperature of $T_{\text{rh}} \simeq 10^{10}$ GeV, one obtains that $f_{\text{NL}} \sim 10^{-60}$ for the modes of cosmological interest (i.e. for k such that $k/a_0 \simeq H_0$), a value which is completely unobservable. This confirms and quantifies our result that, in the case of single field inflation, the epoch of preheating does not alter the amplitude of the scalar bi-spectrum generated during inflation [81]. However, it is worthwhile to add that, while the amplitude of the above non-Gaussianity parameter f_{NL} is small, it seems to be strongly scale dependent.

We believe that a couple of points require further emphasis at this stage of our discussion. Recall that, to fall within the first instability band during preheating, the modes need to satisfy the condition (20). But, in order to neglect the term involving k^2 in the differential equation (16), the modes of interest are actually required to satisfy the condition (21). As we have emphasized earlier, evidently, the condition (21) will be easily satisfied by the large scale modes that already lie within the instability band and thereby satisfying the condition (22). Therefore, it is important to appreciate the fact that the conclusions we have arrived at above apply to all cosmologically relevant scales.

V. DISCUSSION

In this work, we have analyzed the effects of preheating on the primordial bi-spectrum in inflationary models involving a single canonical scalar field. We have illustrated that, certain contributions to the bi-spectrum, such as those due to the combination of the fourth and the seventh terms and that due to the second term, vanish identically at late times. Further, assuming the inflationary potential to be quadratic around its minimum, we have shown that the remaining contributions to the bi-spectrum are completely insignificant during the epoch of preheating when the scalar field is oscillating at the bottom of the potential immediately after inflation. It is important to appreciate the fact that the results we have arrived at apply to any single field inflationary potential that has a parabolic shape near the minimum.

A couple of other points also need to be stressed regarding the conclusions we have arrived at. The results we have obtained supplement the earlier results wherein it has been shown that the power spectrum generated during inflation remains unaffected during the epoch of preheating (see, Ref. [29]; in this context, also see Ref. [82]). Moreover, our results are in support of earlier

discussions which had pointed to the fact that the contributions to the correlation functions at late times will be insignificant if the interaction terms in the actions at the cubic and the higher orders depend on either a time or a spatial derivative of the curvature perturbation [83–87].

Broadly, our effort needs to be extended in two different directions. Firstly, it is important to confirm that the conclusions we have arrived at hold true for potentials which behave differently, say, quartically, near the minima. Further, the exercise needs to be repeated for models involving non-canonical scalar fields [88–90]. In this context, it is worth mentioning that the generalization of the conserved quantity \mathcal{R}_k in the Dirac-Born-Infeld case has been shown to stay constant in amplitude on scales larger than the sonic horizon, a property which allows us to propagate the spectrum from horizon exit till the beginning of the radiation dominated era [89]. Secondly, as we had mentioned, preheating is followed by an epoch of reheating when the energy from the inflaton is expected to be transferred to radiation. It will be interesting to examine the evolution of the bi-spectrum during reheating. However, in order to achieve reheating, the scalar field needs to be coupled to radiation. It is clear that the formalism for evaluating the bi-spectrum involving just the inflaton is required to be extended to a situation wherein radiation too is present and is also coupled to the scalar field.

However, possibly, the most interesting direction opened up by our work concerns multi-field inflation and associated non-Gaussianities [91–96]. Unlike single field models wherein the curvature perturbation associated with the large scale modes is conserved at late times, such a behavior is not necessarily true in multi-field inflation. When many fields are present, the entropy (i.e. the iso-curvature) fluctuations can cause the evolution of curvature perturbations even on super-Hubble scales. Further, in the case of multi-field inflation, it is known that the two-point correlation function can be affected by metric preheating [77]. In other words, the power spectrum calculated at the end of multi-field inflation is not necessarily the power spectrum observed in, say, the CMB data because the post-inflationary dynamics (that is to say preheating instabilities) can modify it. In the same manner, it is tempting to conjecture that the scalar bi-spectrum calculated at the end of multi-field inflation will not necessarily be the same as the one observed in the data [97]. In particular, the so-called consistency relations [43], which relate the three-point and the four-point correlation functions (or, equivalently, the corresponding dimensionless non-Gaussianity parameters f_{NL} and τ_{NL}) might receive corrections in a multi-field context due to metric preheating. We are currently investigating such issues.

- [2] S. Dodelson, *Modern Cosmology* (Academic Press, Elsevier Science, 2003).
- [3] V. Mukhanov, *Physical Foundations of Cosmology* (Cambridge University Press, 2005).
- [4] S. Weinberg, *Cosmology* (Oxford University Press, 2008).
- [5] R. Durrer, *The Cosmic Microwave Background* (Cambridge University Press, 2008).
- [6] P. Peter and J. Uzan, *Cosmologie primordiale*, Échelles (Paris) (Belin, 2005).
- [7] D. H. Lyth and A. R. Liddle, *The primordial density perturbation: Cosmology, inflation and the origin of structure* (Cambridge University Press, 2009).
- [8] H. Kodama and M. Sasaki, Prog. Theor. Phys. Suppl. **78**, 1 (1984).
- [9] V. F. Mukhanov, H. Feldman, and R. H. Brandenberger, Phys. Rept. **215**, 203 (1992).
- [10] J. E. Lidsey, A. R. Liddle, E. W. Kolb, E. J. Copeland, T. Barreiro, et al., Rev. Mod. Phys. **69**, 373 (1997), astro-ph/9508078.
- [11] D. H. Lyth and A. Riotto, Phys. Rept. **314**, 1 (1999), hep-ph/9807278.
- [12] B. A. Bassett, S. Tsujikawa, and D. Wands, Rev. Mod. Phys. **78**, 537 (2006), astro-ph/0507632.
- [13] A. Riotto, pp. 317–413 (2002), hep-ph/0210162.
- [14] W. H. Kinney (2009), 0902.1529.
- [15] D. Baumann (2009), 0907.5424.
- [16] L. Sriramkumar (2009), 0904.4584.
- [17] J. Martin, Braz. J. Phys. **34**, 1307 (2004), astro-ph/0312492.
- [18] J. Martin, Lect. Notes Phys. **669**, 199 (2005), hep-th/0406011.
- [19] J. Martin, Lect. Notes Phys. **738**, 193 (2008), 0704.3540.
- [20] A. D. Linde, Lect. Notes Phys. **738**, 1 (2008), 0705.0164.
- [21] M. S. Turner, Phys. Rev. **D28**, 1243 (1983).
- [22] A. Albrecht, P. J. Steinhardt, M. S. Turner, and F. Wilczek, Phys. Rev. Lett. **48**, 1437 (1982).
- [23] J. Martin and C. Ringeval, JCAP **0608**, 009 (2006), astro-ph/0605367.
- [24] J. Martin and C. Ringeval, Phys. Rev. **D82**, 023511 (2010), 1004.5525.
- [25] D. I. Podolsky, G. N. Felder, L. Kofman, and M. Peloso, Phys. Rev. **D73**, 023501 (2006), hep-ph/0507096.
- [26] J. H. Traschen and R. H. Brandenberger, Phys. Rev. **D42**, 2491 (1990).
- [27] Y. Shtanov, J. H. Traschen, and R. H. Brandenberger, Phys. Rev. **D51**, 5438 (1995), hep-ph/9407247.
- [28] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. **D56**, 3258 (1997), hep-ph/9704452.
- [29] F. Finelli and R. H. Brandenberger, Phys. Rev. Lett. **82**, 1362 (1999), hep-ph/9809490.
- [30] K. Jedamzik, M. Lemoine, and J. Martin, JCAP **1009**, 034 (2010), 1002.3039.
- [31] K. Jedamzik, M. Lemoine, and J. Martin, JCAP **1004**, 021 (2010), 1002.3278.
- [32] R. Easther, R. Flauger, and J. B. Gilmore, JCAP **1104**, 027 (2011), 1003.3011.
- [33] A. Gangui, F. Lucchin, S. Matarrese, and S. Mollerach, Astrophys. J. **430**, 447 (1994), astro-ph/9312033.
- [34] A. Gangui, Phys. Rev. **D50**, 3684 (1994), astro-ph/9406014.
- [35] A. Gangui and J. Martin, Mon. Not. Roy. Astron. Soc. (1999), astro-ph/9908009.
- [36] A. Gangui and J. Martin, Phys. Rev. **D62**, 103004 (2000), astro-ph/0001361.
- [37] J. M. Maldacena, JHEP **0305**, 013 (2003), astro-ph/0210603.
- [38] D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005), astro-ph/0503692.
- [39] X. Chen, Phys. Rev. **D72**, 123518 (2005), astro-ph/0507053.
- [40] X. Chen, M.-x. Huang, S. Kachru, and G. Shiu, JCAP **0701**, 002 (2007), hep-th/0605045.
- [41] D. Langlois, S. Renaux-Petel, D. A. Steer, and T. Tanaka, Phys. Rev. Lett. **101**, 061301 (2008), 0804.3139.
- [42] D. Langlois, S. Renaux-Petel, D. A. Steer, and T. Tanaka, Phys. Rev. **D78**, 063523 (2008), 0806.0336.
- [43] X. Chen, Adv. Astron. **2010**, 638979 (2010), 1002.1416.
- [44] X. Chen, R. Easther, and E. A. Lim, JCAP **0804**, 010 (2008), 0801.3295.
- [45] X. Chen, R. Easther, and E. A. Lim, JCAP **0706**, 023 (2007), astro-ph/0611645.
- [46] S. Hotchkiss and S. Sarkar, JCAP **1005**, 024 (2010), 0910.3373.
- [47] S. Hannestad, T. Haugbolle, P. R. Jarnhus, and M. S. Sloth, JCAP **1006**, 001 (2010), 0912.3527.
- [48] R. Flauger and E. Pajer, JCAP **1101**, 017 (2011), 1002.0833.
- [49] P. Adshead, W. Hu, C. Dvorkin, and H. V. Peiris, Phys. Rev. **D84**, 043519 (2011), 1102.3435.
- [50] P. Adshead, C. Dvorkin, W. Hu, and E. A. Lim, Phys. Rev. **D85**, 023531 (2012), 1110.3050.
- [51] J. Martin and L. Sriramkumar, JCAP **1201**, 008 (2012), 1109.5838.
- [52] D. K. Hazra, L. Sriramkumar, and J. Martin (2012), 1201.0926.
- [53] A. Gangui, J. Martin, and M. Sakellariadou, Phys. Rev. **D66**, 083502 (2002), astro-ph/0205202.
- [54] R. Holman and A. J. Tolley, JCAP **0805**, 001 (2008), 0710.1302.
- [55] W. Xue and B. Chen, Phys. Rev. **D79**, 043518 (2009), 0806.4109.
- [56] P. D. Meerburg, J. P. van der Schaar, and P. S. Corasaniti, JCAP **0905**, 018 (2009), 0901.4044.
- [57] X. Chen, JCAP **1012**, 003 (2010), 1008.2485.
- [58] E. Komatsu and D. N. Spergel, Phys. Rev. **D63**, 063002 (2001), astro-ph/0005036.
- [59] E. Komatsu, D. N. Spergel, and B. D. Wandelt, Astrophys. J. **634**, 14 (2005), astro-ph/0305189.
- [60] D. Babich and M. Zaldarriaga, Phys. Rev. **D70**, 083005 (2004), astro-ph/0408455.
- [61] M. Liguori, F. Hansen, E. Komatsu, S. Matarrese, and A. Riotto, Phys. Rev. **D73**, 043505 (2006), astro-ph/0509098.
- [62] C. Hikage, E. Komatsu, and T. Matsubara, Astrophys. J. **653**, 11 (2006), astro-ph/0607284.
- [63] J. Fergusson and E. P. Shellard, Phys. Rev. **D76**, 083523 (2007), astro-ph/0612713.
- [64] A. P. Yadav, E. Komatsu, and B. D. Wandelt, Astrophys. J. **664**, 680 (2007), about SDSS, astro-ph/0701921.
- [65] P. Creminelli, L. Senatore, and M. Zaldarriaga, JCAP **0703**, 019 (2007), astro-ph/0606001.
- [66] A. P. Yadav and B. D. Wandelt, Phys. Rev. Lett. **100**, 181301 (2008), 0712.1148.
- [67] C. Hikage, T. Matsubara, P. Coles, M. Liguori, F. K. Hansen, et al., Mon. Not. Roy. Astron. Soc. **389**, 1439 (2008), 0802.3677.

- [68] O. Rudjord, F. K. Hansen, X. Lan, M. Liguori, D. Marinucci, et al., *Astrophys. J.* **701**, 369 (2009), 0901.3154.
- [69] K. M. Smith, L. Senatore, and M. Zaldarriaga, *JCAP* **0909**, 006 (2009), 0901.2572.
- [70] J. Smidt, A. Amblard, C. T. Byrnes, A. Cooray, A. Heavens, et al., *Phys. Rev.* **D81**, 123007 (2010), 1004.1409.
- [71] J. Fergusson, M. Liguori, and E. Shellard (2010), 1006.1642.
- [72] M. Liguori, E. Sefusatti, J. Fergusson, and E. Shellard, *Adv. Astron.* **2010**, 980523 (2010), 1001.4707.
- [73] A. P. Yadav and B. D. Wandelt, *Adv. Astron.* **2010**, 565248 (2010), 1006.0275.
- [74] E. Komatsu, *Class. Quant. Grav.* **27**, 124010 (2010), 1003.6097.
- [75] D. Larson, J. Dunkley, G. Hinshaw, E. Komatsu, M. Nolta, et al., *Astrophys. J. Suppl.* **192**, 16 (2011), 1001.4635.
- [76] E. Komatsu et al. (WMAP Collaboration), *Astrophys. J. Suppl.* **192**, 18 (2011), 1001.4538.
- [77] F. Finelli and R. H. Brandenberger, *Phys. Rev.* **D62**, 083502 (2000), hep-ph/0003172.
- [78] C. Bender and S. Orszag, *Advanced Mathematical Methods for Scientists and Engineers: Asymptotic Methods and Perturbation Theory* (Springer, 1978).
- [79] M. Abramowitz and I. A. Stegun, *Handbook of mathematical functions with formulas, graphs, and mathematical tables* (National Bureau of Standards, Washington, US, 1970), ninth ed.
- [80] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products* (Academic Press, New York and London, 1965).
- [81] K. Kohri, D. H. Lyth, and C. A. Valenzuela-Toledo, *JCAP* **1002**, 023 (2010), 0904.0793.
- [82] A. R. Liddle, D. H. Lyth, K. A. Malik, and D. Wands, *Phys. Rev.* **D61**, 103509 (2000), hep-ph/9912473.
- [83] S. Weinberg, *Phys. Rev.* **D72**, 043514 (2005), hep-th/0506236.
- [84] S. Weinberg, *Phys. Rev.* **D74**, 023508 (2006), hep-th/0605244.
- [85] S. Weinberg, *Phys. Rev.* **D78**, 123521 (2008), 0808.2909.
- [86] S. Weinberg, *Phys. Rev.* **D79**, 043504 (2009), 0810.2831.
- [87] S. Weinberg, *Phys. Rev.* **D83**, 063508 (2011), 1011.1630.
- [88] L. Lorenz, J. Martin, and C. Ringeval, *Phys. Rev.* **D78**, 063543 (2008), 0807.2414.
- [89] L. Lorenz, J. Martin, and C. Ringeval, *Phys. Rev.* **D78**, 083513 (2008), 0807.3037.
- [90] J. Lachapelle and R. H. Brandenberger, *JCAP* **0904**, 020 (2009), 0808.0936.
- [91] G. I. Rigopoulos, E. P. S. Shellard, and B. J. W. van Tent, *Phys. Rev.* **D73**, 083522 (2006), astro-ph/0506704.
- [92] D. Seery and J. E. Lidsey, *JCAP* **0509**, 011 (2005), astro-ph/0506056.
- [93] F. Arroja, S. Mizuno, and K. Koyama, *JCAP* **0808**, 015 (2008), 0806.0619.
- [94] X. Gao, *JCAP* **0806**, 029 (2008), 0804.1055.
- [95] S. Mizuno, F. Arroja, and K. Koyama, *Phys. Rev.* **D80**, 083517 (2009), 0907.2439.
- [96] S. Renaux-Petel, S. Mizuno, and K. Koyama, *JCAP* **1111**, 042 (2011), 1108.0305.
- [97] K. Enqvist, A. Jokinen, A. Mazumdar, T. Multamaki, and A. Vaihkonen, *Phys. Rev. Lett.* **94**, 161301 (2005), astro-ph/0411394.